Interest Rate Terminology

The terms 'nominal' and 'effective' enter into consideration when the compounding period (i.e. interest period) is *less than one year*.

A *nominal interest rate(r)* is obtained by multiplying an interest rate that is expressed over a short time period by the number of interest periods in a longer time period:

nominal rate, r = interest rate per period x no. of periods

For example, if i = 1% per month, the nominal rate per year , r , is (1)(12) = 12%/yr

Since nominal rates are essentially simple interest rates, they *cannot* be used in any of the interest formulas (effective rates must be used, as calculated below).

Effective rates can be obtained from nominal rates via the following formula:

$$i = (1 + r/m)^m - 1$$ where m = no times interest is comp'd

For example, if i = 1% per month, effective i/yr = $(1 + 0.12/12)^{12} - 1 = 12.68\%$

Interest Rate Statements

There are 3 general ways to express interest rates as shown below:

	Interest Rate Statement	Comment
(1)	i = 12% per month i = 12% per year	When no compounding period is given, rate is *effective*
(2)	i = 10% per year, comp'd semiannually i= 3% per quarter, comp'd monthly	When compounding period is given and it is not the same as period of interest rate, it is *nominal*
(3)	i = effective 10%/yr, comp'd semiannually i = effective 4% per quarter, comp'd monthly	When compounding period is given and rate is specified as effective, rate *is effective* over stated period

Effective Interest Calculations

Nominal rates can be converted into effective rates via the following equation:

$$i = (1 + r / m)^m - 1$$

Where : i = effective interest rate for any time period
r = nominal rate for same time period as i
m = no. times interest is comp'd in period specified for i

Example: For an interest rate of 1.2% per month, determine the nominal and effective rates (a) per quarter, and (b) per year

Solution:

(a) Nominal r / quarter = (1.2)(3) = 3.6% per quarter

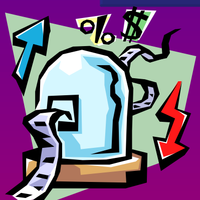
Effective i / quarter = $(1 + 0.036/3)^3 - 1$ = 3.64% per quarter

(b) Nominal i /yr = (1.2)(12) = 14.4% per year

Effective i / yr = $(1 + 0.144/12)^{12} - 1$ = 15.39% per year

Single Amounts With PP>CP

For problems involving single amounts, the payment period(PP) is usually longer than the comp'd period(CP). For these problems, there are an infinite number of i and n combinations that can be used, with only two restrictions:



(1) The i must be an *effective* interest rate, and

(2) The time units on n must be *the same* as those of i (if i is a rate per quarter, then n must be the no. of quarters between P and F)

Example: How much money will be in an account in 5 years if $10,000 is deposited now at an interest rate of 1% per month? Use three different rates: (a) monthly, (b) quarterly , and (c) yearly.

(a) For monthly rate, 1% is effective:

F = 10,000(F/P,1%,60) = $18,167

(b) For a quarterly rate, effective i/quarter = $(1 + 0.03/3)^3 - 1$ = 3.03%

F = 10,000(F/P,3.03%,20) = $18,167

(c) For an annual rate, effective i/yr = $(1 + 0.12/12)^{12} - 1$ = 12.68%

F = 10,000(F/P,12.68%,5) = $18,165

Series With PP>=CP

For series cash flows, *first step* is to determine relationship between PP and CP

When PP>=CP, the *only* procedure(2 steps) that can be used is as follows:

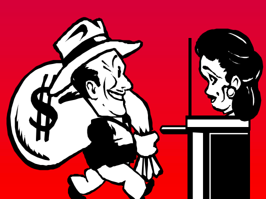
(1) First, find effective i per PP (ex: if PP is quarters, *must* find effective *i/quarter*

(2) Then, determine n, where n is equal to the no. of A values involved (ex: quarterly payments for six years yields n = 24)

Example: How much money will be accumulated in 10 years from a deposit of $500 every 6 months if the interest rate is 1% per month?

Solution: Since PP>CP, first step is to find effective i per PP (six months):

$i/6 \text{ mos.} = (1 + 0.06/6)^6 - 1 = 6.15\%$

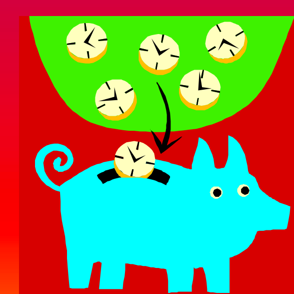


Next step is to determine n:

$n = 10(2) = 20$

Now, $F = 500(F/A,6.15\%,20)$

$= \$18,692$ (Excel)



Cash Flow With PP<CP

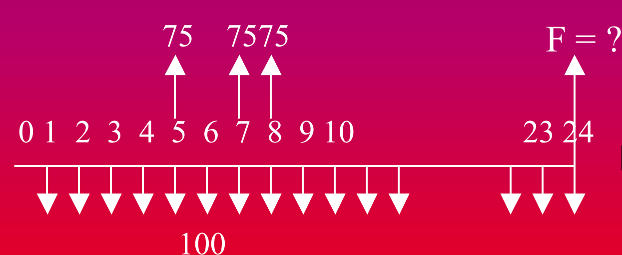
When PP<CP, no interperiod compounding is assumed. Therefore, *withdrawals* are moved *to beginning of interest period* in which they occur and *deposits to end*.

[This condition (i.e. PP<CP) is the *only time* the actual cash flow diagram is changed]

Example: A person deposits $100 per month into a savings account for 2 years. If $75 is withdrawn in months 5,7 and 8 (in addition to the deposits), how much will be in the account after 2 years at i = 6% per year, comp'd quarterly.

Solution: Since PP<CP, the cash flow diagram must be changed as follows:

from this



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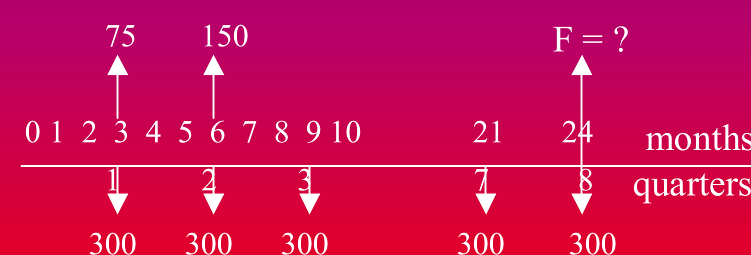

F = 300(F/A,1.5%,8) – 75(F/P,1.5%,7) - 150 (F/P,1.5%,6)

= $2,283

Continuous Compounding

When the interest period is infinitely small, interest is *comp'd continuously*

For continuous compounding, equation is: $i = e^r - 1$

PP>CP

Example: If a person deposits $500 into an account every 3 months for five years at an interest rate of 6% per year compounded continuously, how much will be in the account at the end of that time?

Solution: Nominal rate,r, per three months is 1.5%. Therefore,

Effective i/ 3 months = $e^{0.015} - 1 = 1.51\%$

F = 500(F/A,1.51%,20)
= $11,573 (Excel)

Varying Interest Rates

When interest rates vary over time, use the interest rates associated with their respective time periods to find P.

Example: Find the present worth of a uniform series of $2500 deposits in years 1 thru 8 if the interest rate is 7% for the first five years and 10% per year thereafter.

Solution:

$$P = 2{,}500(P/A,7\%,5) + 2{,}500(P/A,10\%,3)(P/F,7\%,5)$$
$$= \$14{,}683$$

An equivalent AW value can be obtained by replacing each cash flow amount with 'A' and setting the equation equal to the calculated P value as follows:

$$14{,}683 = A(P/A,7\%,5) + A(P/A,10\%,3)(P/F,7\%,5)$$
$$A = \$2500$$